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A Photon Free Method to Solve Radiation Transport Equations

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**A Photon Free Method to Solve Radiation
Transfer Equations**

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Abstract

- The multi-group discrete-ordinate equations of radiation transfer is solved for the **first time** by Newton's method.
- It is a photon free method because the photon variables are eliminated from the radiation equations to yield a $N_{\text{group}} \times N_{\text{direction}}$ **smaller** but **equivalent** system of equations.
- The smaller set of equations can be solved more **efficiently** than the original set of equations.
- Newton's method is **more stable** than the Semi-implicit Linear method currently used by conventional radiation codes

The Backward Euler, Multi-group, Discrete-ordinate equations to be solved are

$$\left(\frac{1}{c\Delta t} I + \mu_d D + \sigma_g(T) \right) \psi_{g,d} = \sigma_g(T) B_g(T) + \hat{s}_{g,d} + \frac{\psi_{g,d}^n}{c\Delta t}$$

$$C_p(T) \frac{T - T^n}{\Delta t} = \sum_{g,d} w_d \sigma_g(T) (\psi_{g,d} - B_g(T)) + q$$

- D is a finite difference matrix
- T is the temperature
- $\psi_{g,d}$ is the intensity for group g and direction d
- $B_g(T)$ is the blackbody function
- $C_p(T)$ and $\sigma_g(T)$ is the heat capacity and cross section
- Our goal is solve this system, written as $G(\psi, T) = 0$.
- # unknowns = size(ψ) + size(T) = $(n_x \times n_d \times n_g) + n_x$

Ψ is eliminated from $G(\Psi, T) = 0$ to give the smaller $F(T) = 0$; this improves efficiency.

- Solve the transport equation formally

$$\left(\frac{1}{c\Delta t} I + \mu_d D + \sigma_g(T) \right) \psi_{g,d} = \sigma_g(T) B_g(T) + \hat{s}_{g,d} + \frac{\psi_{g,d}^n}{c\Delta t}$$

- Plug ψ into the temperature equation to get

$$F(T) \equiv C_p(T) \frac{T - T^n}{\Delta t} - \sum_{g,d} w_d \sigma_g(T) \left(H_{g,d}^{-1} \left(\sigma_g(T) B_g(T) + \hat{s}_{g,d} + \frac{\psi_{g,d}^n}{c\Delta t} \right) - B_g(T) \right) - q$$

- # of unknowns = n_x

Conventional deterministic methods do **not solve**
 $G(\Psi, T)=0$, but solve a **linear** approximation.

- The Semi-implicit Linear approximation
 - Lags $C_p^n = C_p(T^n)$ and $\sigma_g^n = \sigma_g(T^n)$, and then
 - **Linearizes**, $B_g(T) = B_g^n + B'_g(T - T^n)$, to give

$$\left(\frac{1}{c\Delta t} I + \mu_d D + \sigma_g^n \right) \psi_{g,d} = \sigma_g^n \left(B_g^n + B'_g (T - T^n) \right) + \hat{s}_{g,d} + \frac{\psi_{g,d}^n}{c\Delta t}$$

$$C_p^n \frac{T - T^n}{\Delta t} = \sum_{g,d} w_d \sigma_g^n \left(\psi_{g,d} - B_g^n - B'_g (T - T^n) \right) + q$$

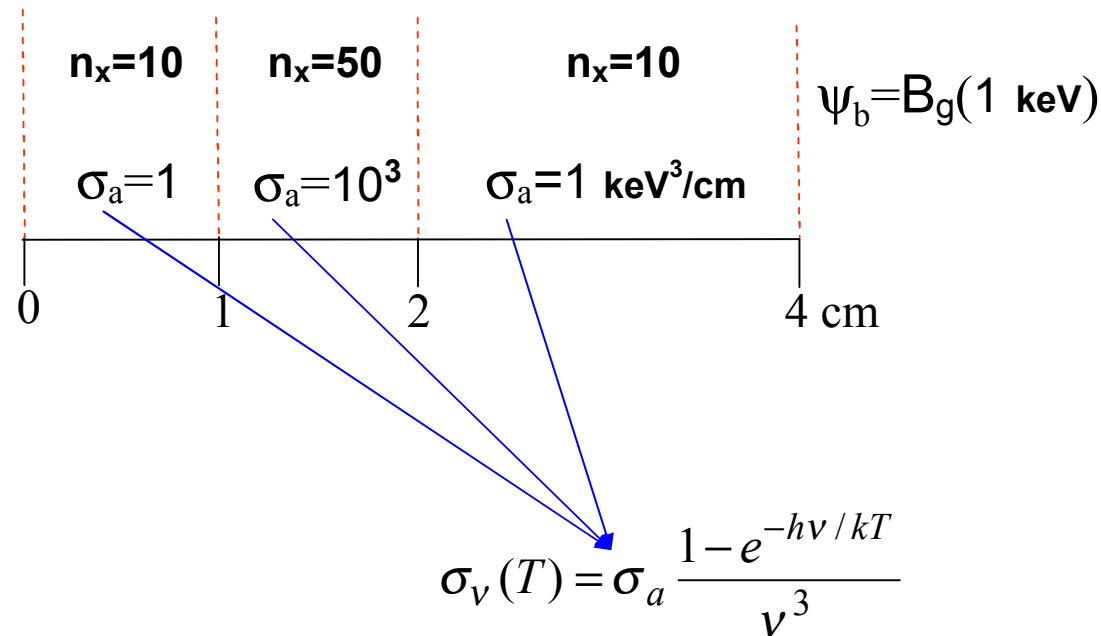
- Eliminating $(T - T^n)$ yields

$$\left(\frac{1}{c\Delta t} I + \mu_d D + \sigma_g^n \right) \psi_{g,d}^{SiL} = \frac{1}{2} \chi_g \eta \sum_{g',d'} w_{d'} \sigma_{g'}^n \psi_{g',d'}^{SiL} + \tilde{s}_{g,d}$$

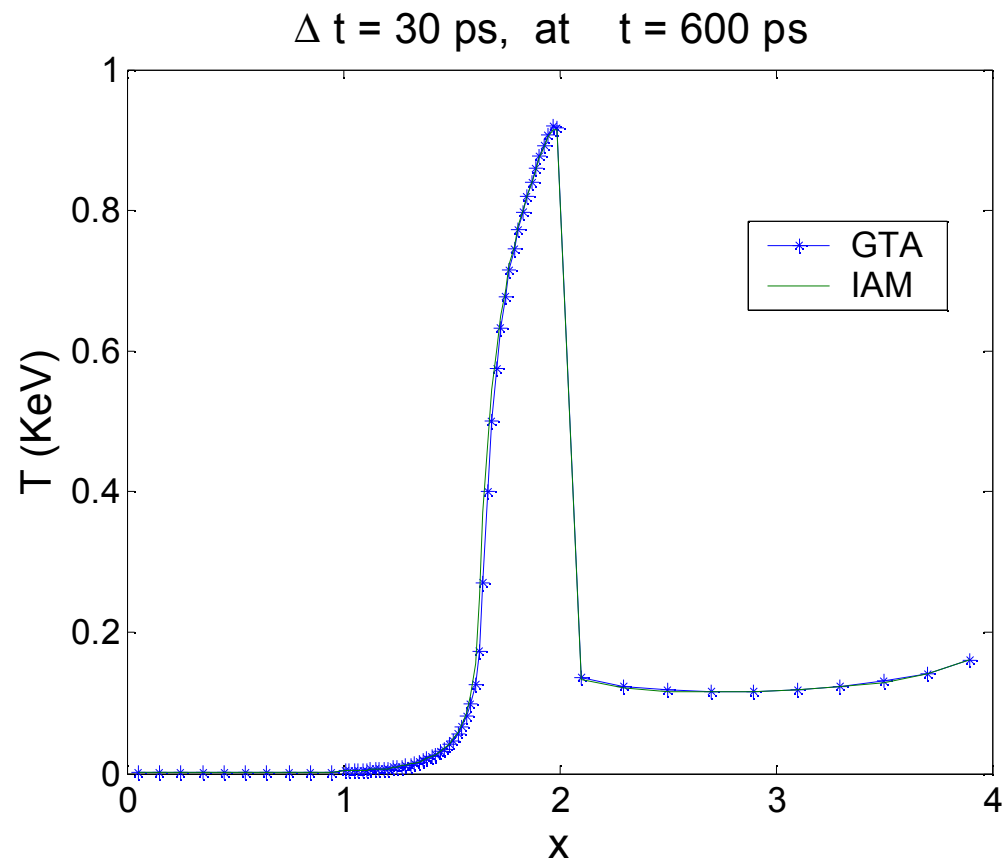
may be negative;
 may yield negative
 ψ^{SiL} and T^{SiL}

Test Problem

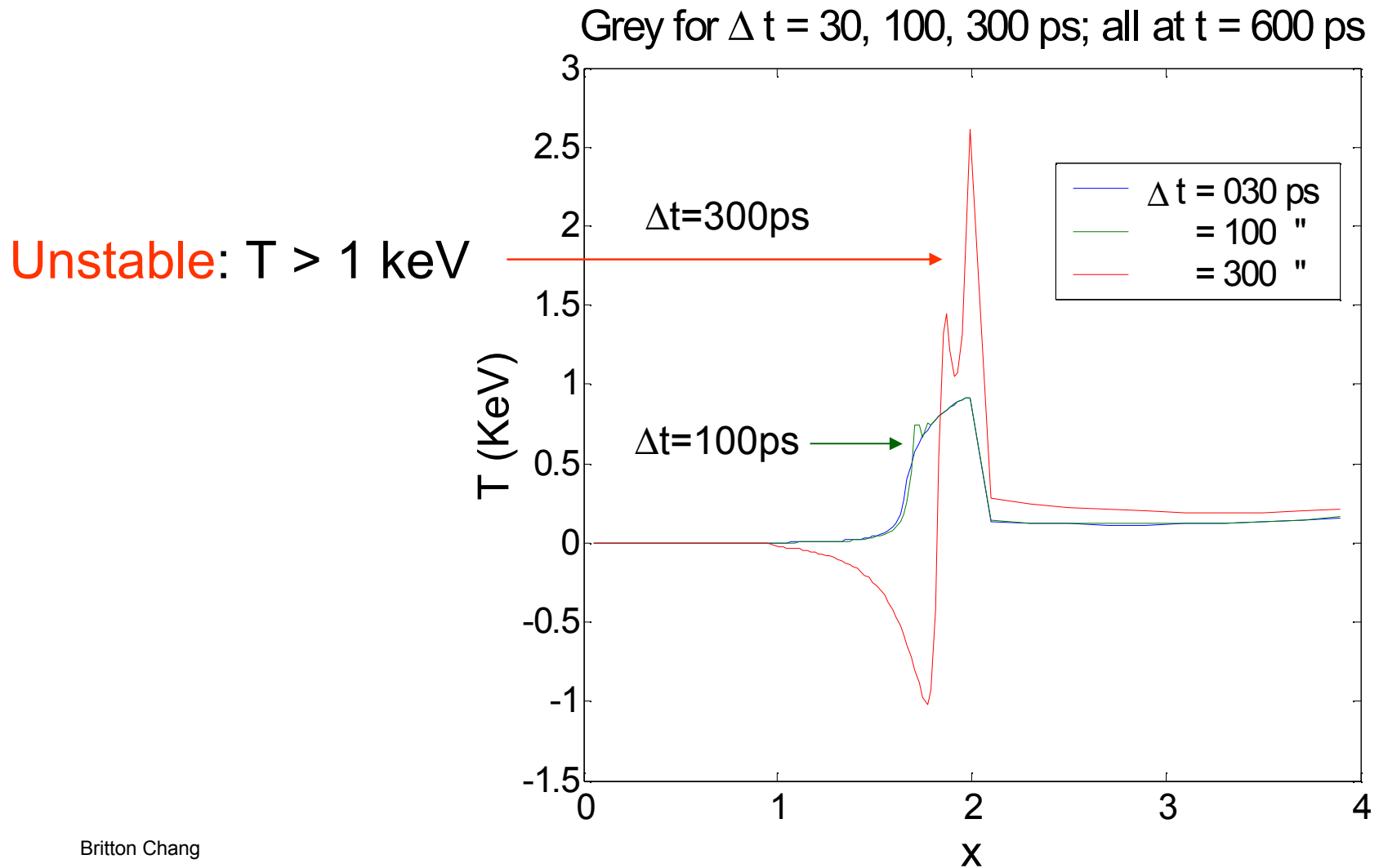
- $C_p = 5 \times 10^{14}$ erg/cm³-keV everywhere
- $N_d = 4$; $N_g = 50$, $h\nu_{\min} = 10^{-5}$ keV, $h\nu_{\max} = 10$ keV



Linear vs. Nonlinear

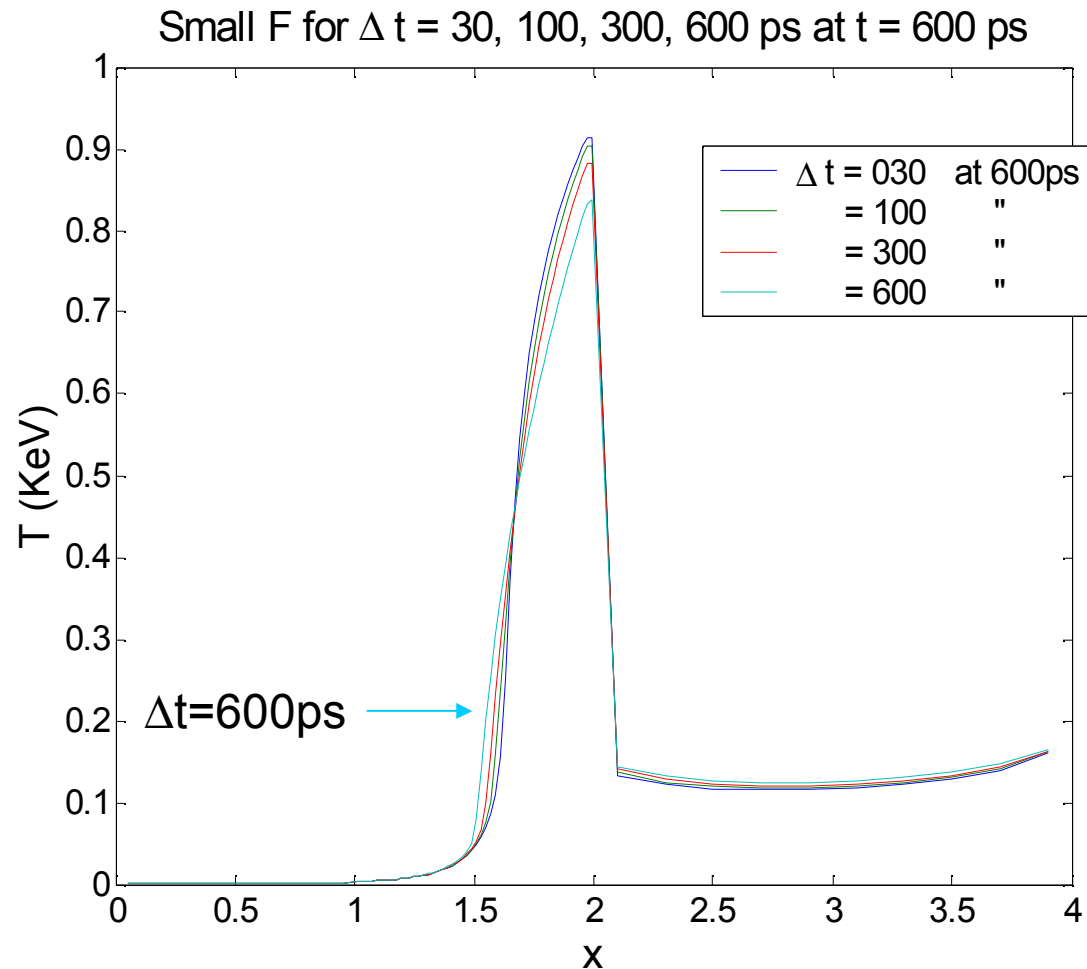


Linear results for increasing Δt



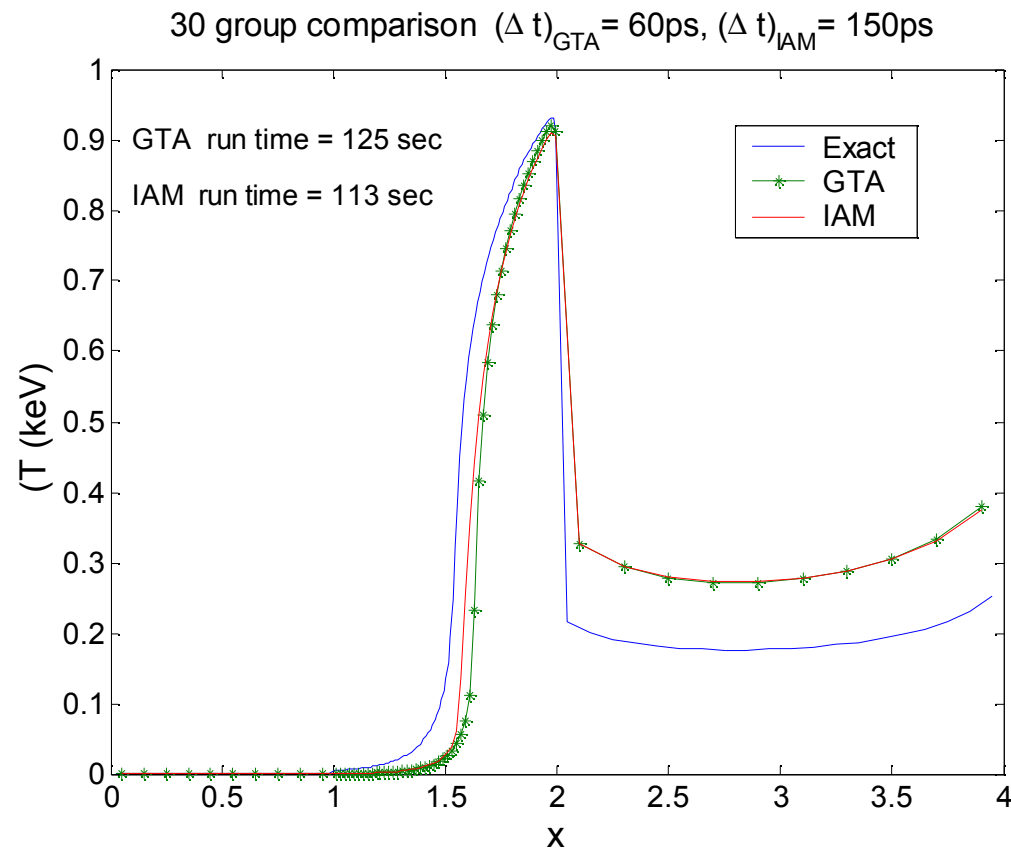
Nonlinear results for increasing Δt

Stable : $T < 1$ keV



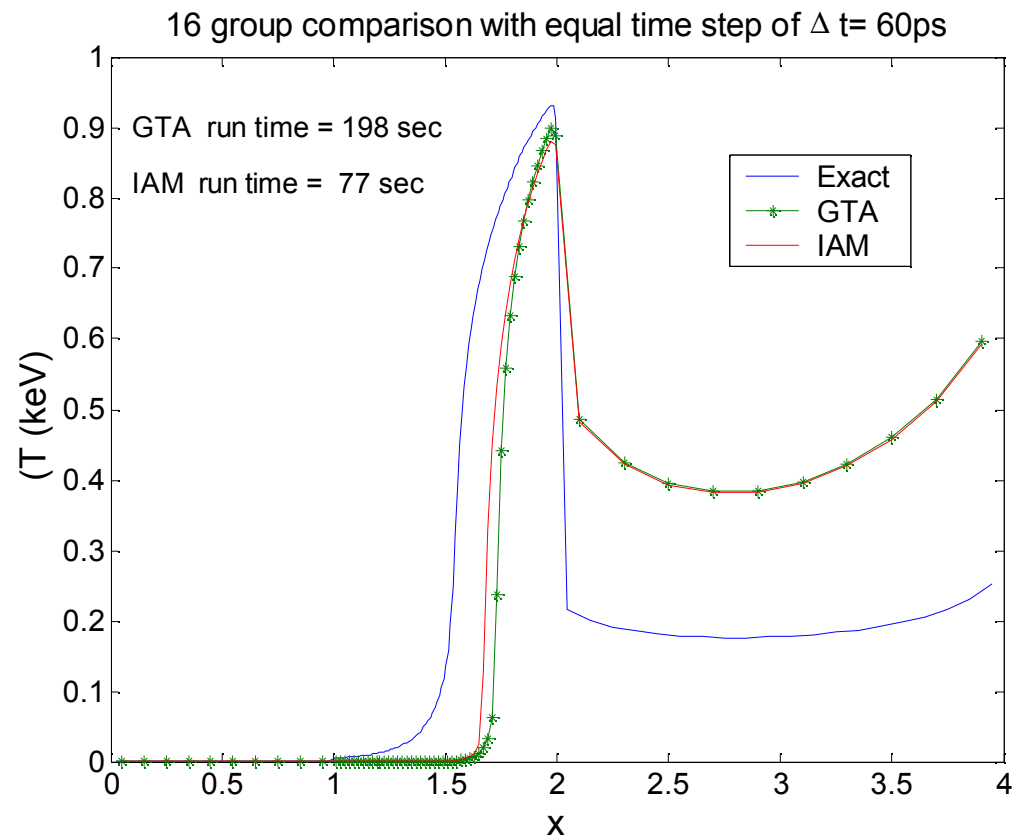
30 group comparison with different time steps

- Time step
 - Lin: 60 ps
 - NL: 150 ps
- Run time
 - Lin: 125 sec
 - NL: 113 sec



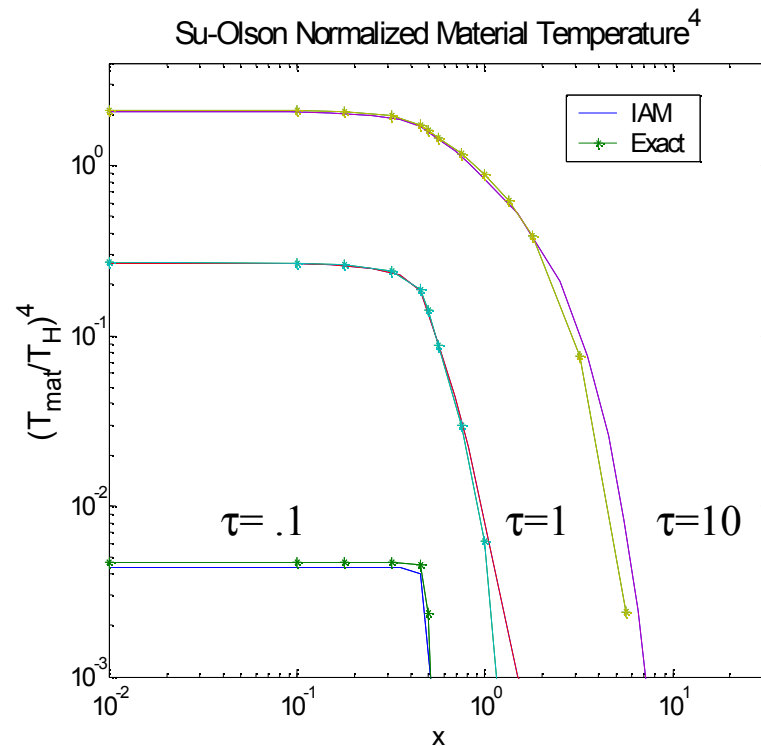
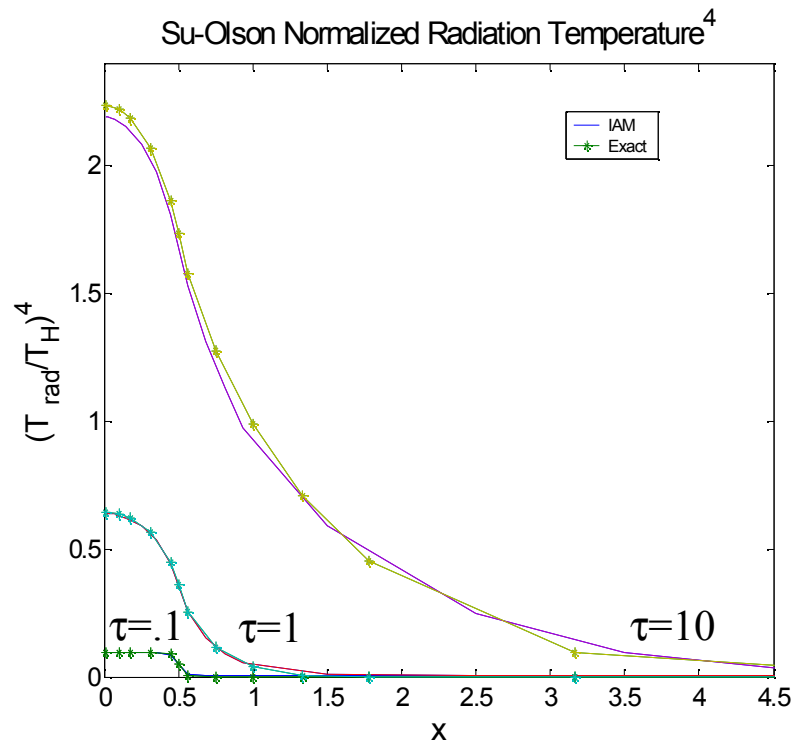
Nonlinear is as fast as linear

- Time step
 - Lin: 60 ps
 - NL: 60 ps
- Run time
 - Lin: 198 sec
 - NL: 77 sec



Nonlinear results for Su-Olson Problem

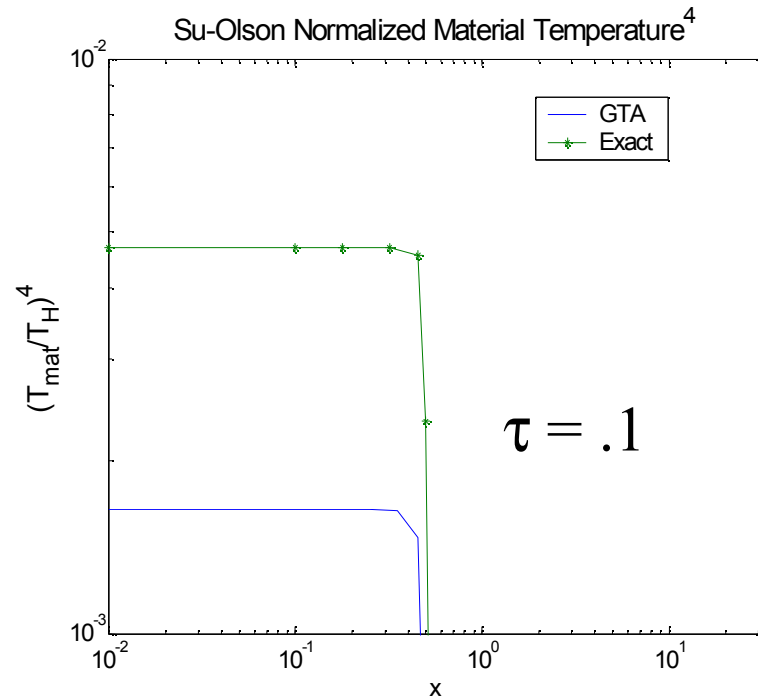
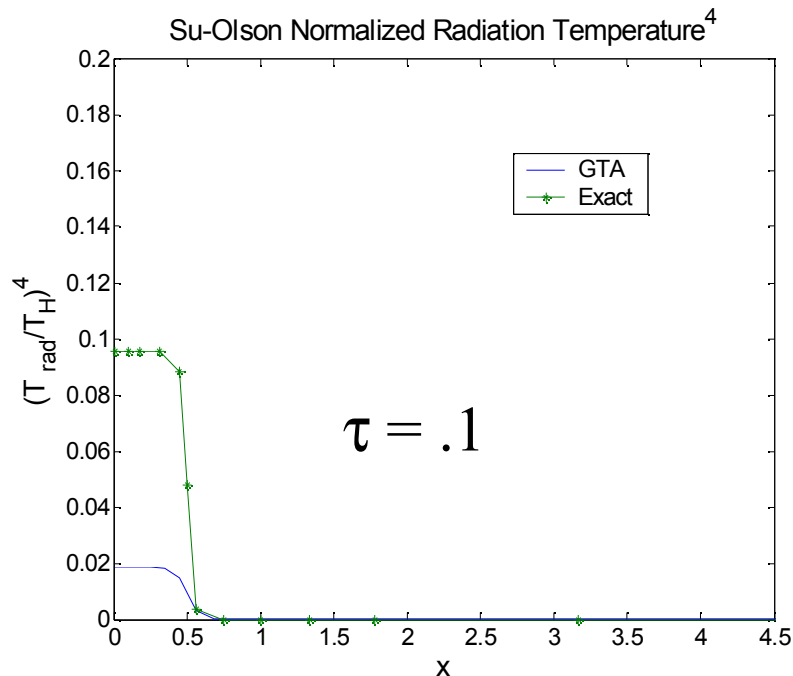
No scattering, Grey, $C_p = 4 a_{\text{rad}} T^3$, $\sigma = 1 \text{ cm}^{-1}$, $n_d = 20$, $n_x = 47$, $\Delta t = 334 \text{ fs}$



Run time = 14 sec for $\tau = .1$

Linear results for Su-Olson Problem

No scattering, Grey, $C_p = 4 a_{\text{rad}} T^3$, $\sigma = 1 \text{ cm}^{-1}$, $n_d=20$, $n_x=47$, $\Delta t = .334\text{fs}$



Run time = 1052 sec

Conclusion

- Newton's method is more stable than linear method
- Nonlinear method can be faster than linear method because the nonlinear method can take larger time steps than the linear method

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